Air Traffic Flow Management for the National Airspace System

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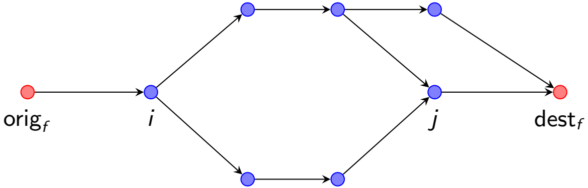
Outline

1. Model formulation
2. Estimating model parameters
3. Performance analysis and preliminary results
4. Conclusions and further work

Model formulation

Bertsimas, Lulli, and Odoni model

- Planes fly between airports and through sectors with defined capacity
- Deterministic and discrete time model
- Produces an optimal assignment of delays to flights
- Flights may be dynamically rerouted to avoid congestion
- Origin-destination routes represented as directed acyclic graph



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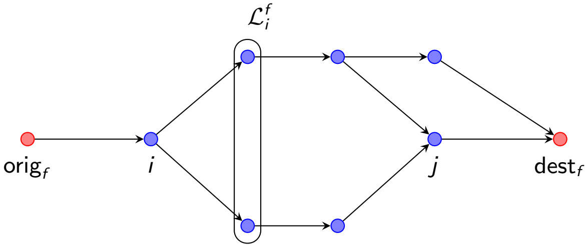

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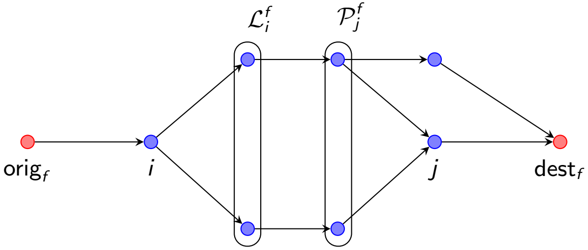

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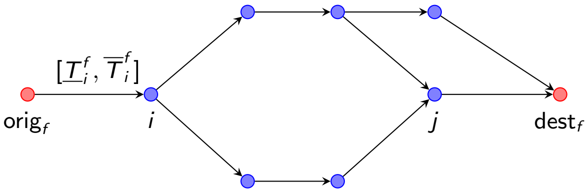

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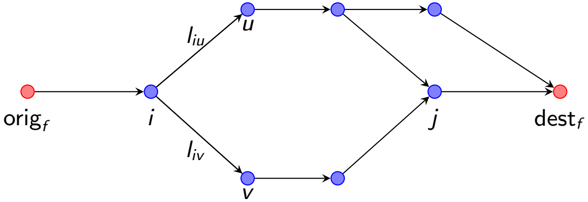
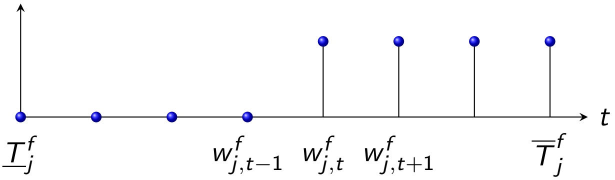


Decision variables

$$w_{j,t}^f = \begin{cases} 1 & \text{if flight } f \text{ arrives at sector } j \text{ by time } t \\ 0 & \text{otherwise} \end{cases}$$

$w_{j,t}^f$ only defined for those sectors j in f's graph, within the feasible time interval $[\underline{T}_j^f, \overline{T}_j^f]$



Constraints

Capacity Constraints

1. # flights arriving at airport k at time t must not exceed $A_k(t)$
2. # flights departing airport k at time t must not exceed $D_k(t)$
3. # flights in sector j at time t must not exceed $S_j(t)$

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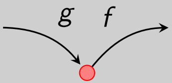
Turnaround time for connecting flights

4. If (g, f) are a pair of connecting flights, flight f cannot depart until s_f minutes after g has arrived



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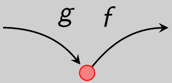
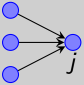


Definition of decision variables

5. If f has arrived at j by time t, it has arrived by time $t + 1$

Sector traversal time

6. A flight cannot arrive at sector j by time t if it has not arrived at a preceding sector $j' \in \mathcal{P}_j$ by time $t - l^f_{j'j}$



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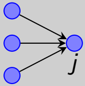

Subsequent sector

7. If flight f has arrived in sector i by $\overline{T}^f_i$, it must arrive in at least one sector $i' \in \mathcal{L}^f_i$ by $\overline{T}^f_{i'}$
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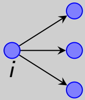

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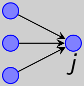

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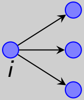


Total flight time

9. The total flight time must not exceed the maximum duration of the flight

Model parameters

- Airport capacities $A_k(t), D_k(t)$ for all airports k, time t
- Sector capacities $S_j(t)$ for all sectors j, times t
- The directed acyclic graphs that describe the flight routes
  - Time intervals: $[\underline{T}_j^f, \overline{T}_j^f]$
  - Time to fly from sector i to sector j: l_{ij}^f
- Maximum flight duration: $\max_f$
- Connecting flight pairs (g, f) and turnaround time s_f

Estimating model parameters

Data sources

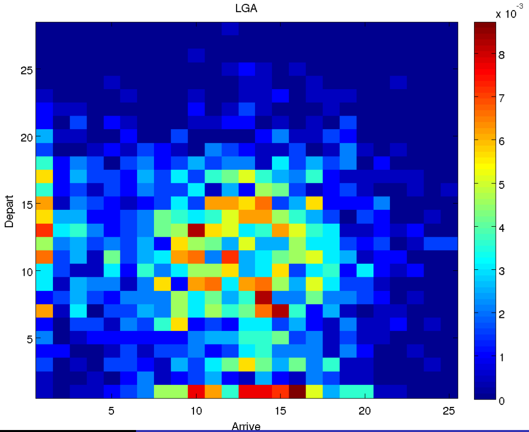
Model requires large amounts of data from multiple sources

Data from John Cho, Richard DeLaura, and Ngaire Underhill:

- ETMS provides sector entrance/exit times for flight graphs
- SDAT provides sector entrance/exit times for flights
- ASPM provides arrival and departure times for flights
- RITA provides delay information and tail numbers (for tracking connecting flights)
- APM provides airport arrival and departure capacities
- Weather-impacted sector capacities from workload model of Cho, Welch, Underhill

Airport capacities

- Estimates from two months of historical APM data
- Number of arrivals and departures in 15 min interval
- Construct estimates for different runway configurations and weather conditions (VMC or IMC)



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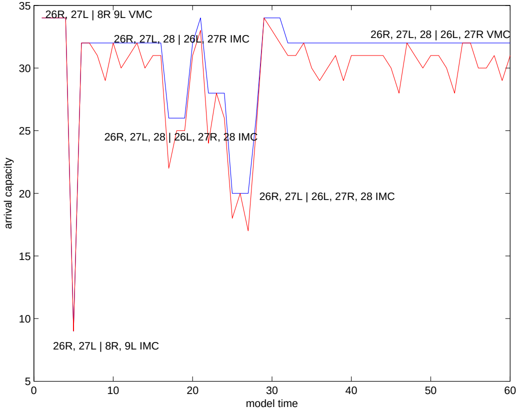
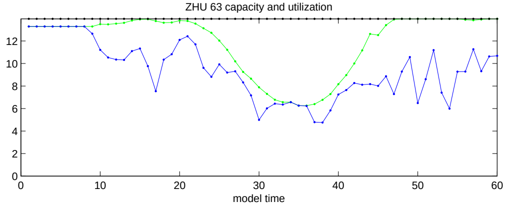
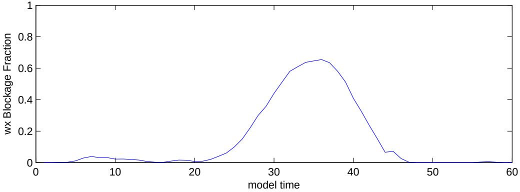


Weather-impacted sector capacities

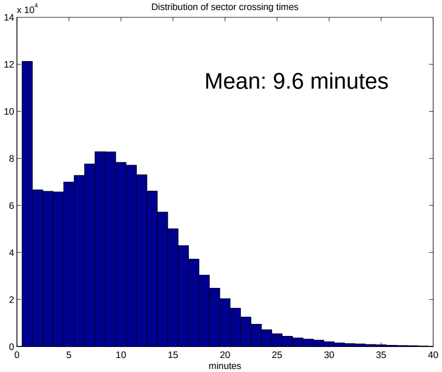
- Use impacted sector capacities from Cho, Welch, Underhill
- Use SDAT data to compute fraction of time spent in sector for each non-model flight
- Lower sector capacities to account for non-model flights





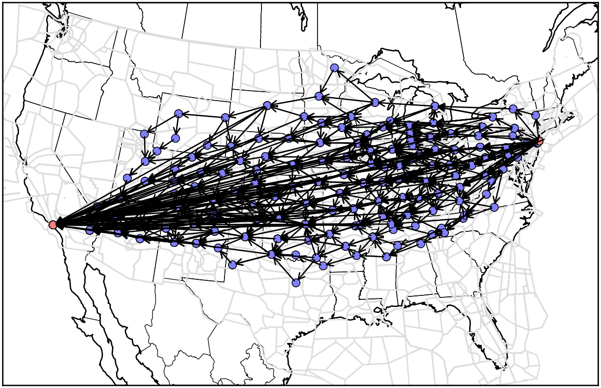
Time spent in sector

- Model uses 15 minute time intervals
- Necessary to obtain airport and sector capacities
- Flights often spend much less than 15 min in a sector
- Drop sectors occupied for less than 8 minutes
- Maintains total flight time (through $[\underline{T}_j^f, \overline{T}_j^f]$, $l_{j'j}^f$)
- But ignores effect on sector capacity



Constructing flight graphs

Use 25 days of ETMS data to construct a graph of sectors traversed by flights (*e.g.* flights from JFK to LAX):



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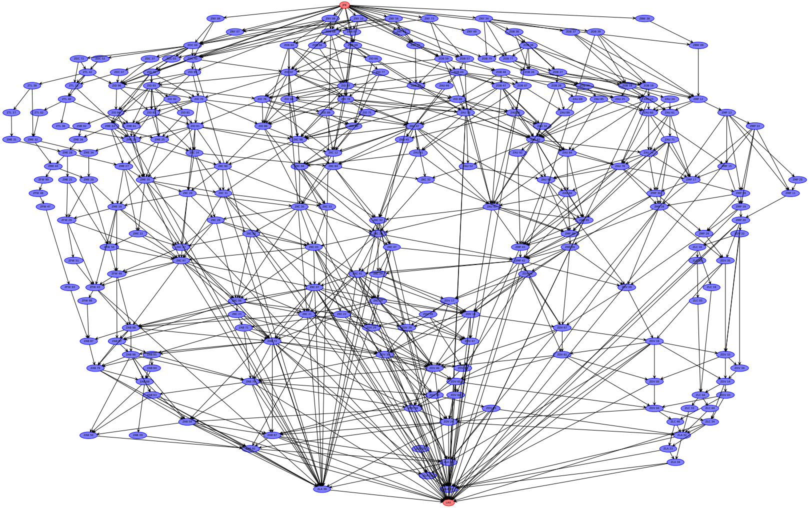
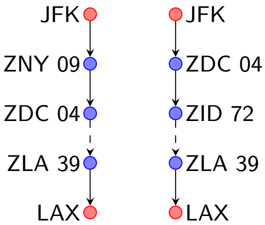


Constructing simple flight graphs

- Need a way to simplify flight graphs
- Consider two different paths from JFK to LAX



- Define a metric on paths:

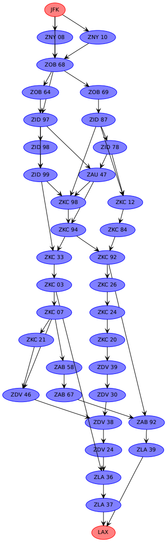
$$d(p_1, p_2) = \max(|p_1|, |p_2|) - |p_1 \cap p_2|$$

- Select K unique paths that share many edges with other paths by solving:

$$\underset{\{\tilde{p}_k\}_{k=1}^{K}}{\text{minimize}} \quad \sum_{k=1}^{K} \sum_{p_j \neq \tilde{p}_k} d(\tilde{p}_k, p_j) \;\; \text{subject to} \;\; \tilde{p}_r \neq \tilde{p}_s, \;\forall r, s$$

Constructing simple flight graphs

Graph is union of K unique paths (*e.g.* JFK to LAX with $K = 10$)



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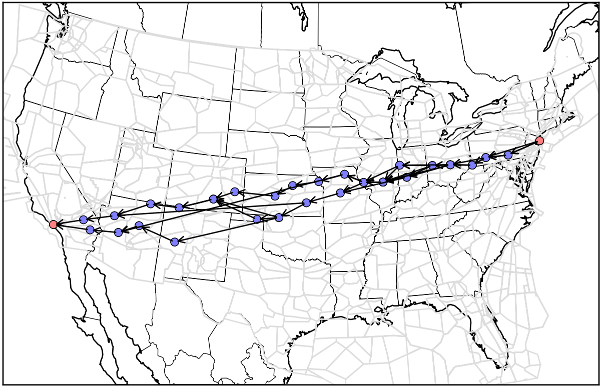


- These paths may lie in a cluster
- Tactical rather than strategic rerouting
- Might prefer graphs with more diverse routes

Performance analysis and preliminary results

Model information

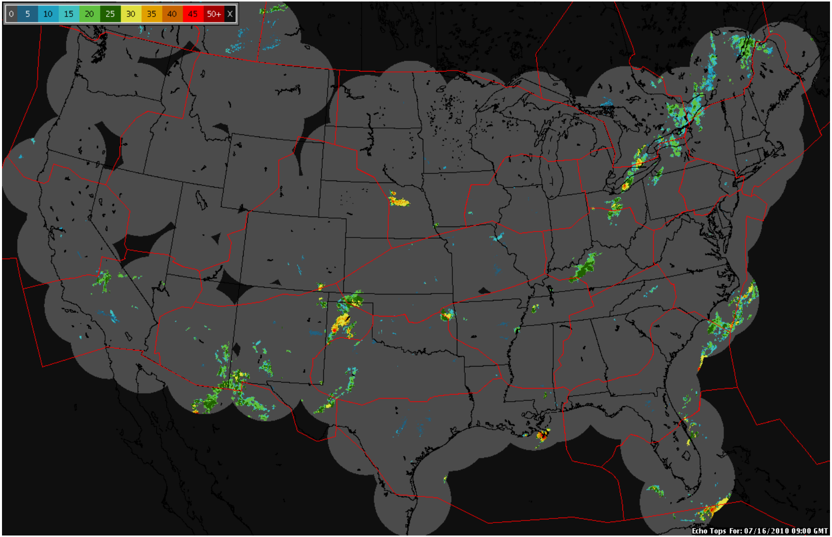
Model includes:

- All (302) high-altitude sectors within continental US
- 130 super-high-altitude sectors (missing ZOA and ZSE)
- OEP 35 largest airports (excluding Honolulu)
- Time frame: 9AM - midnight GMT
- 15 minute time intervals

Model used to analyze July 16th, 2010

- Flight DAGs for 985/1122 airport pairs
- 3590 flights included in the model
- 1123 pairs of connecting flights
- Adjust time intervals $[\underline{T}_j^f, \overline{T}_j^f]$ by scheduled departure time d_f

Echo Tops for July 16th, 2010



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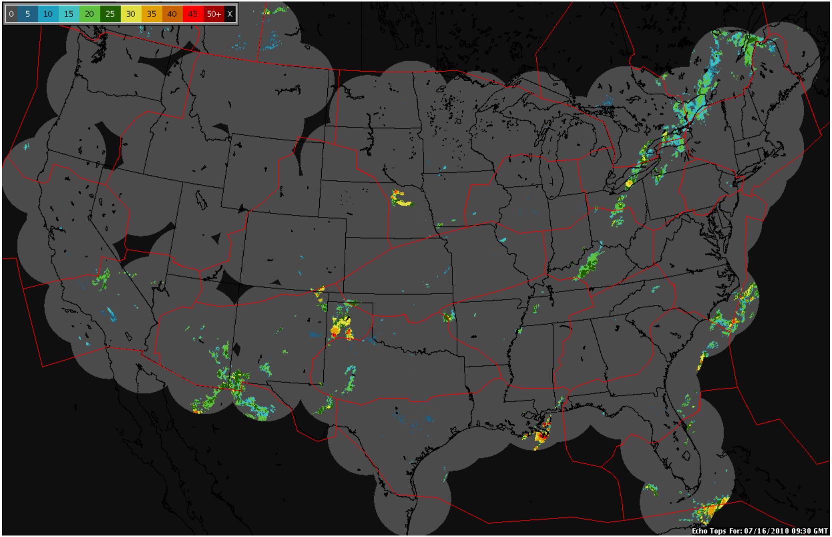

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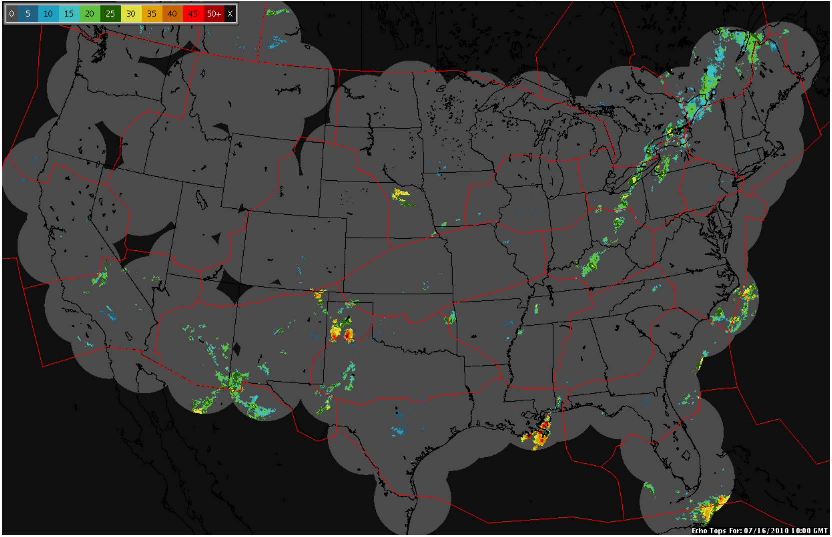

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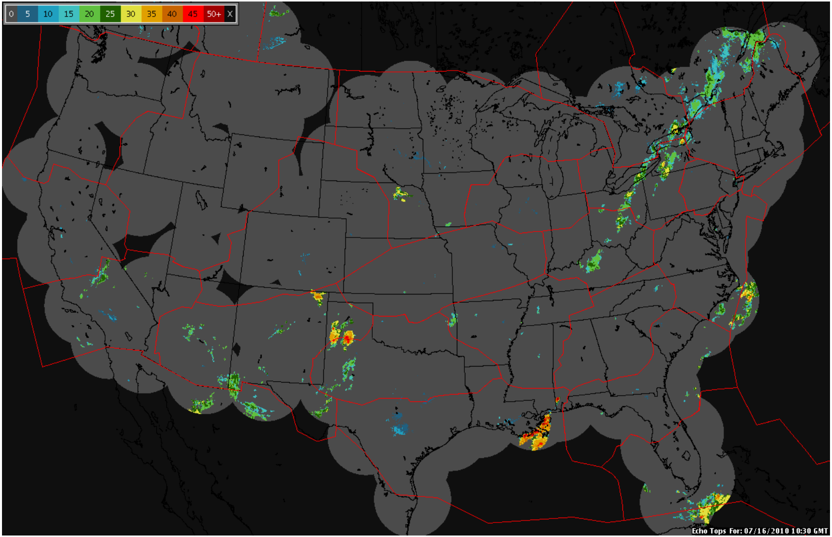

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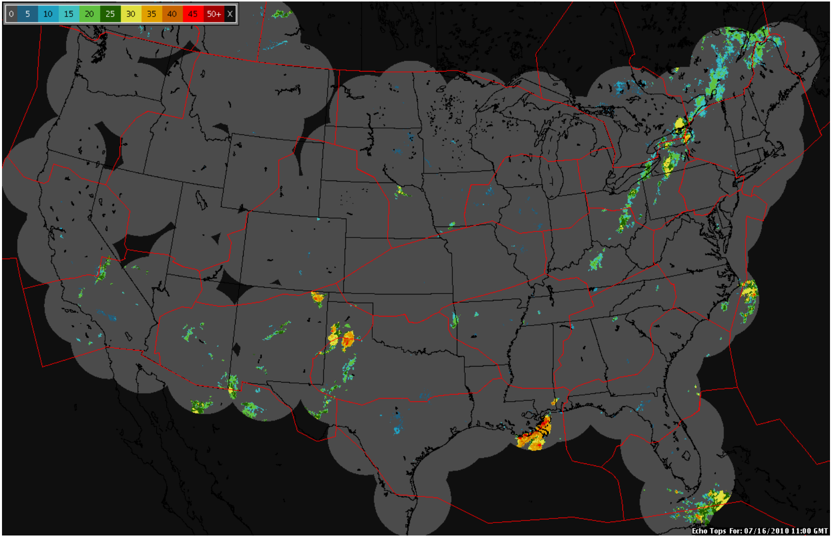

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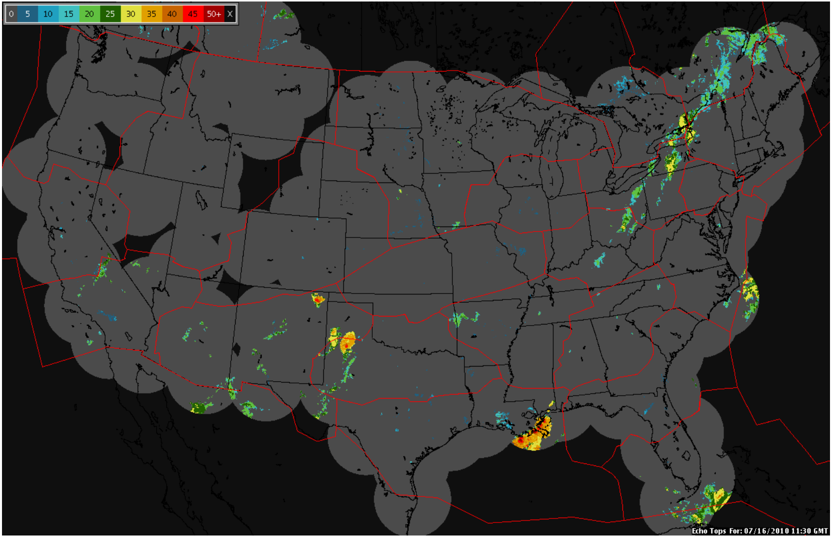

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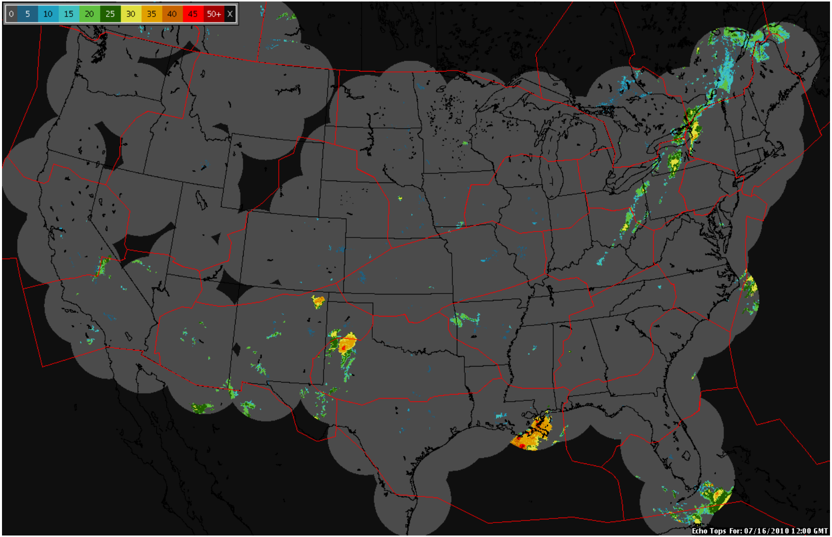

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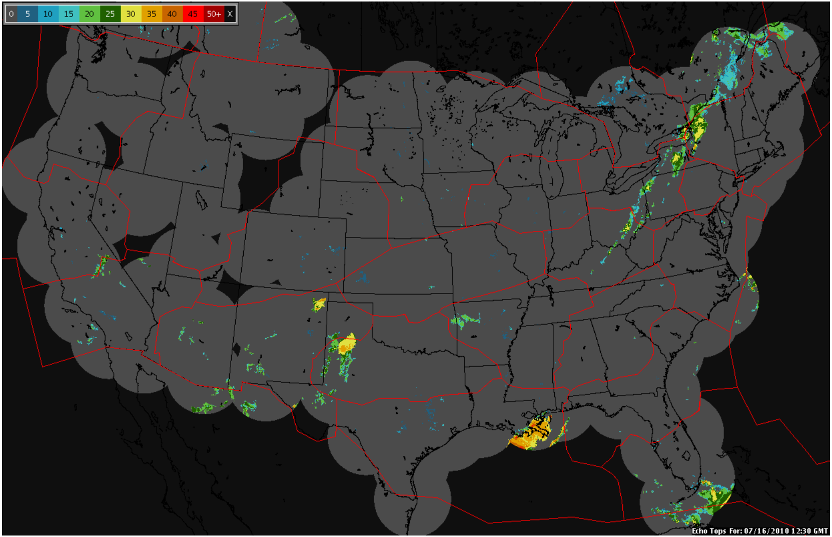

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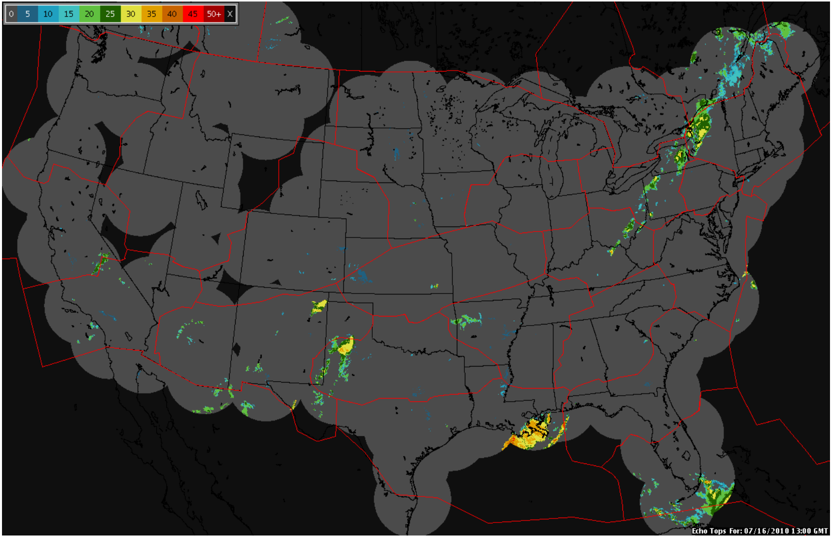

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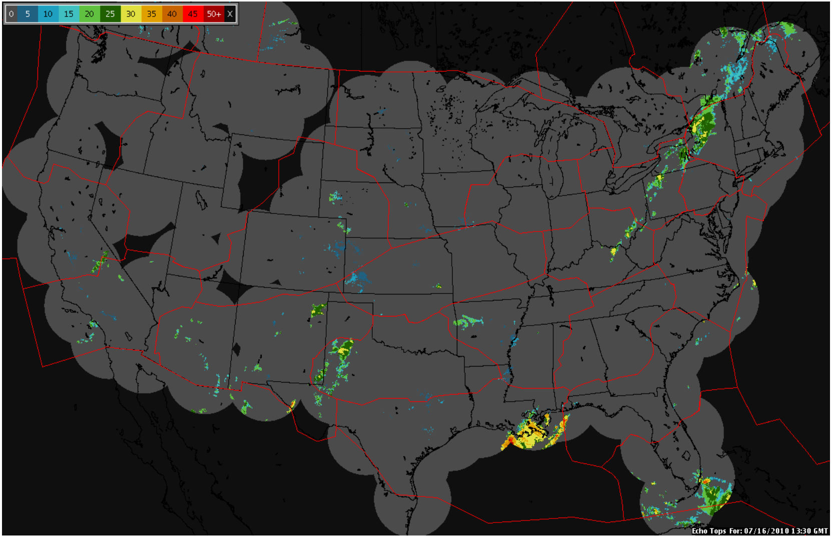

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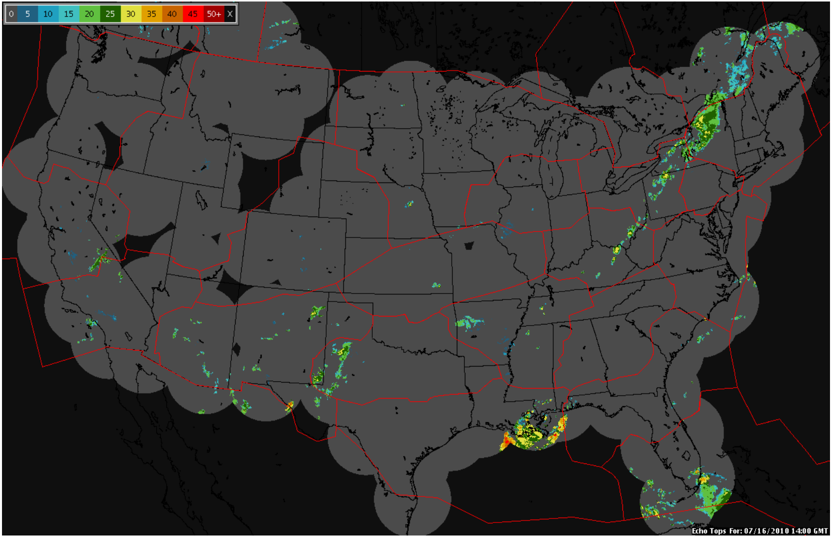

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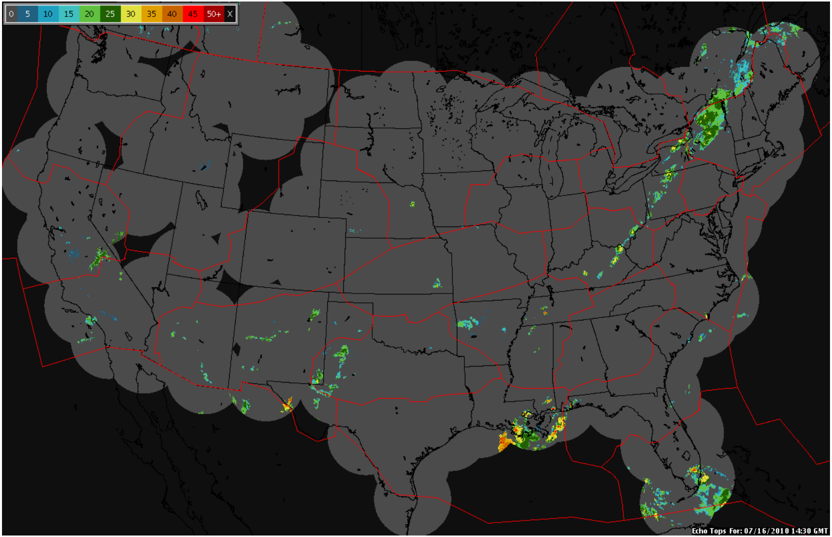

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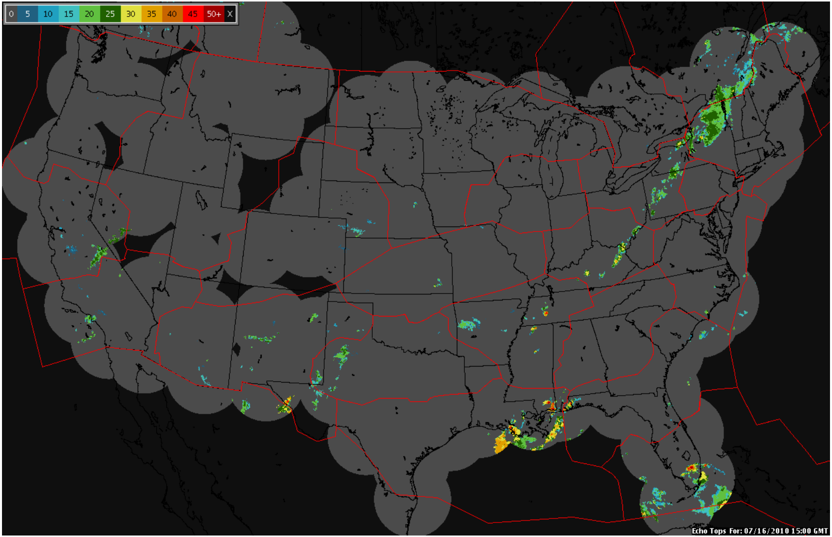

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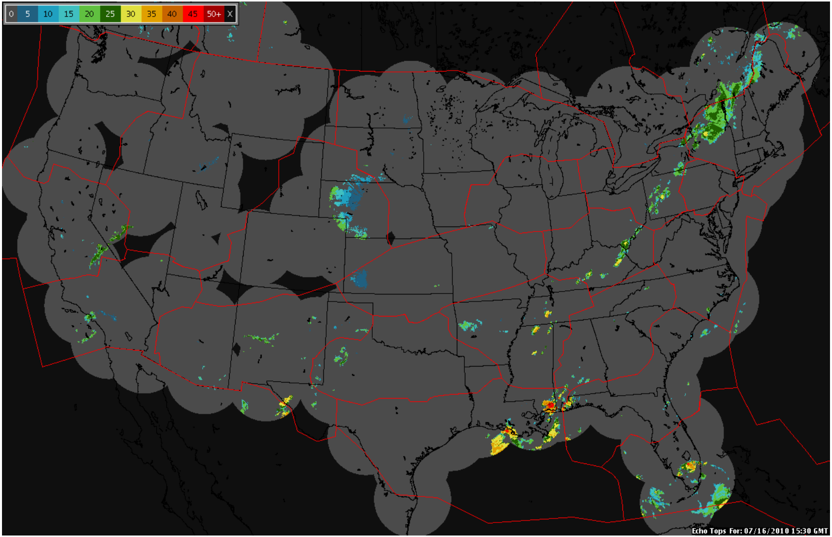

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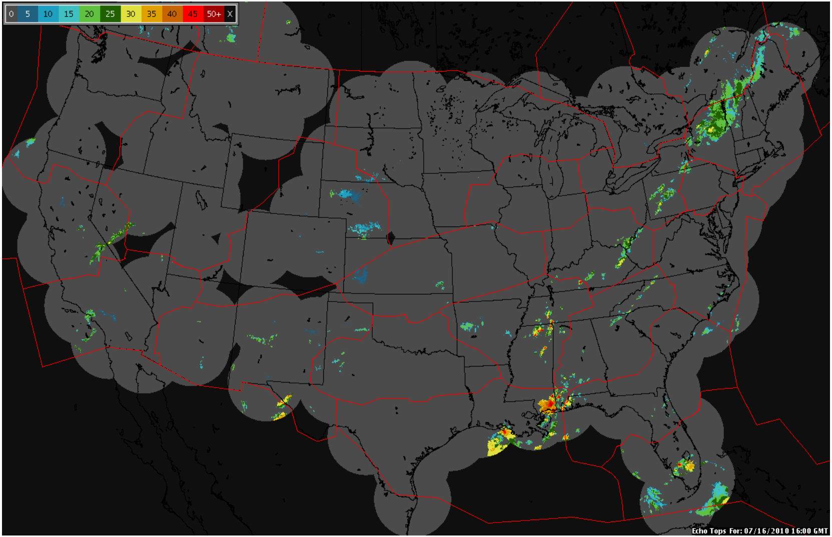

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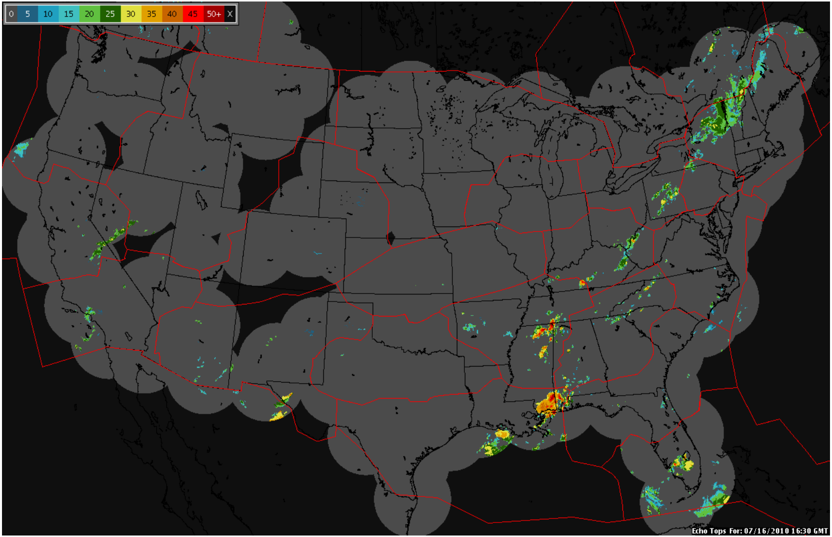

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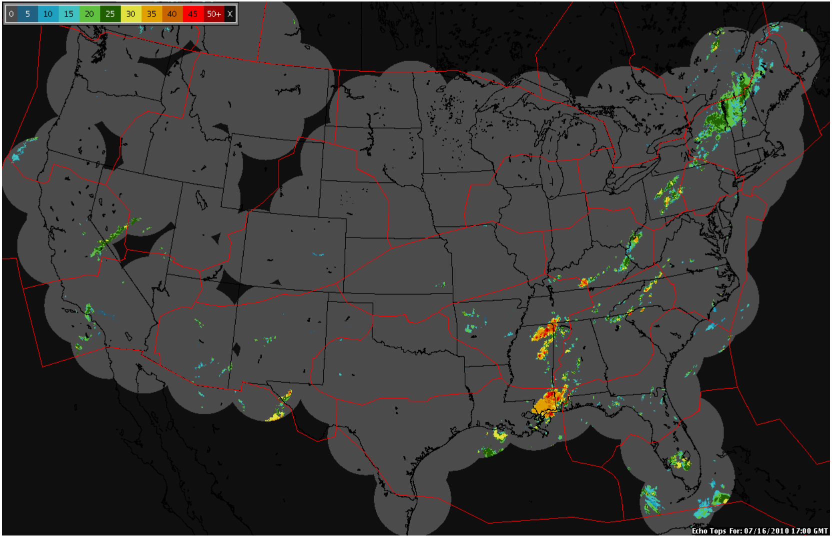

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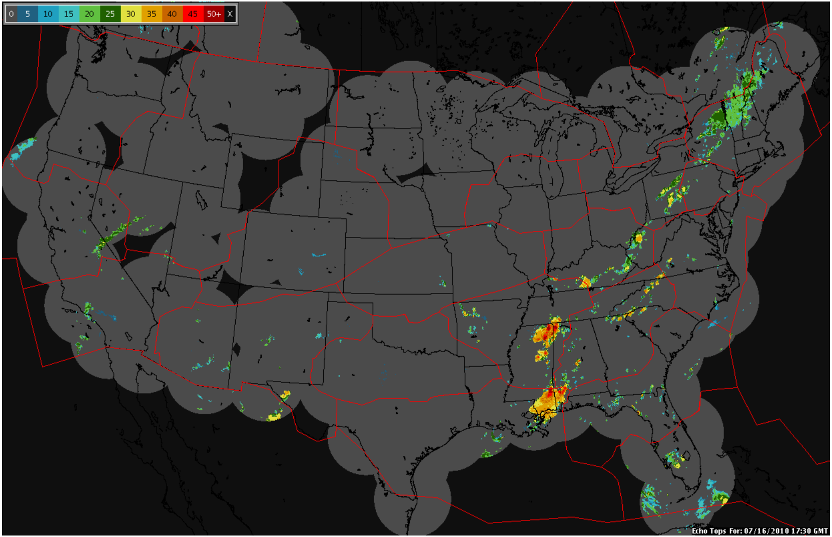

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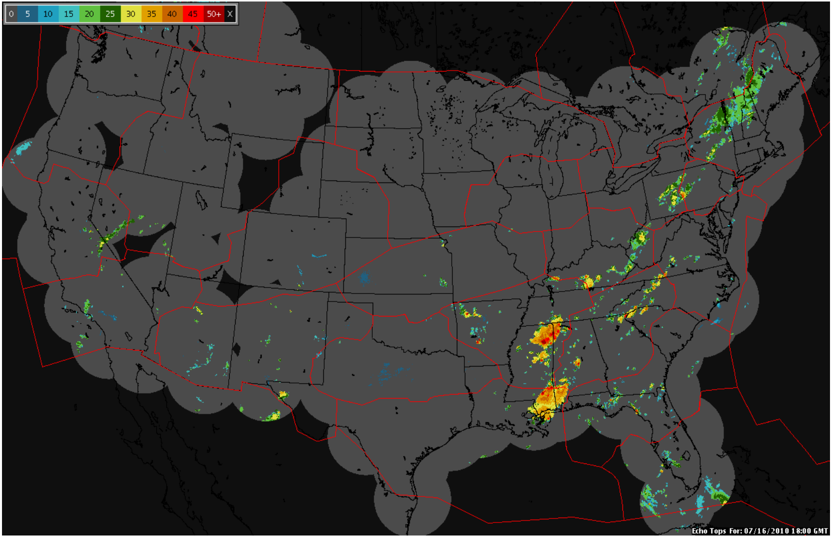

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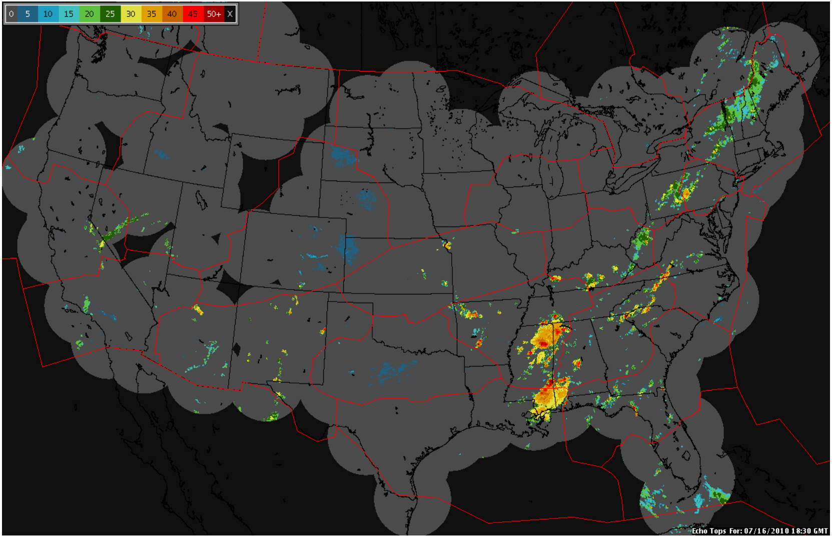

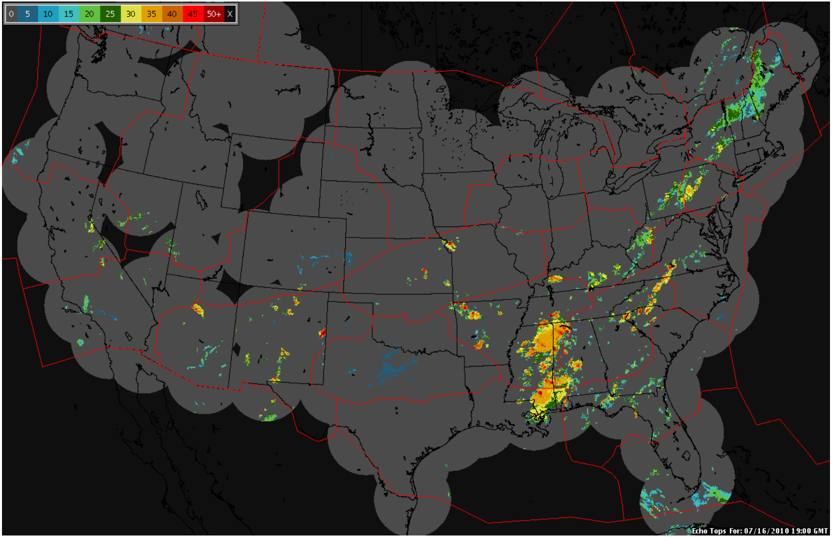
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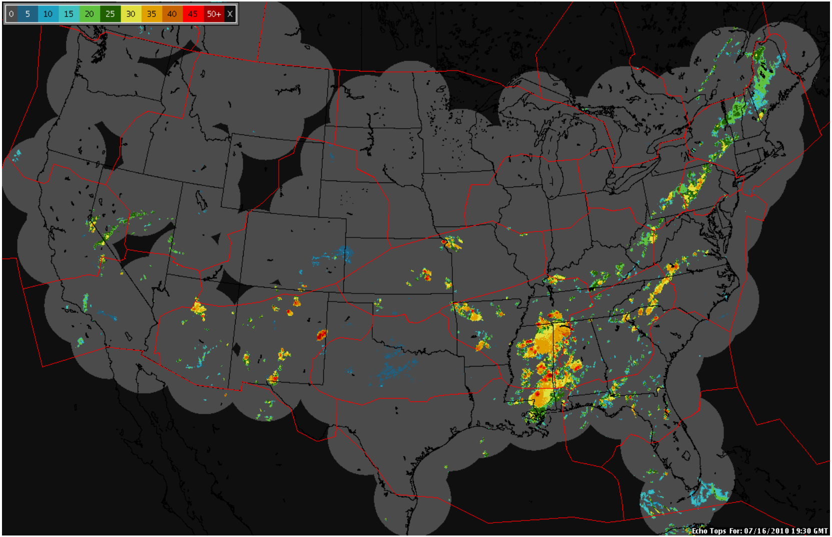

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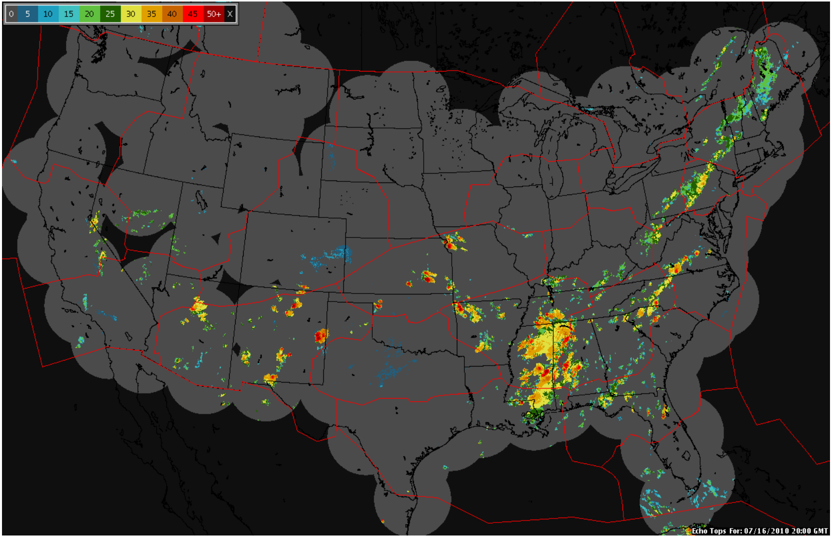

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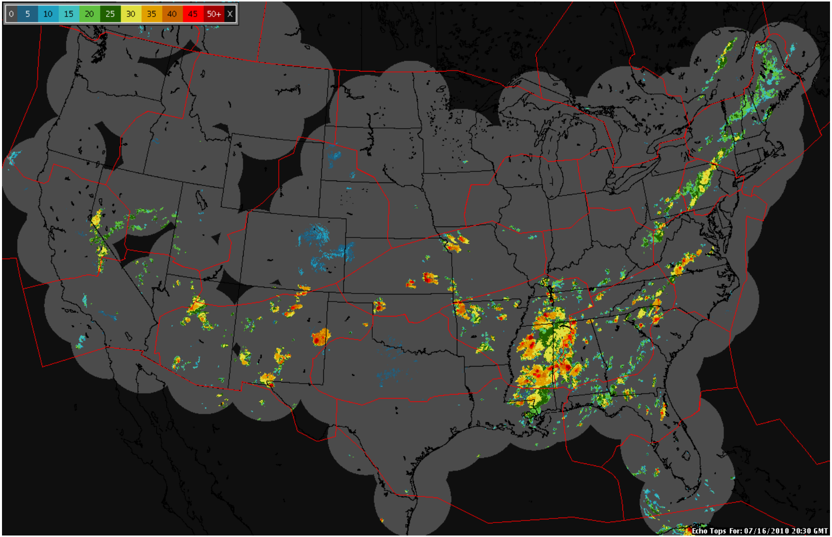

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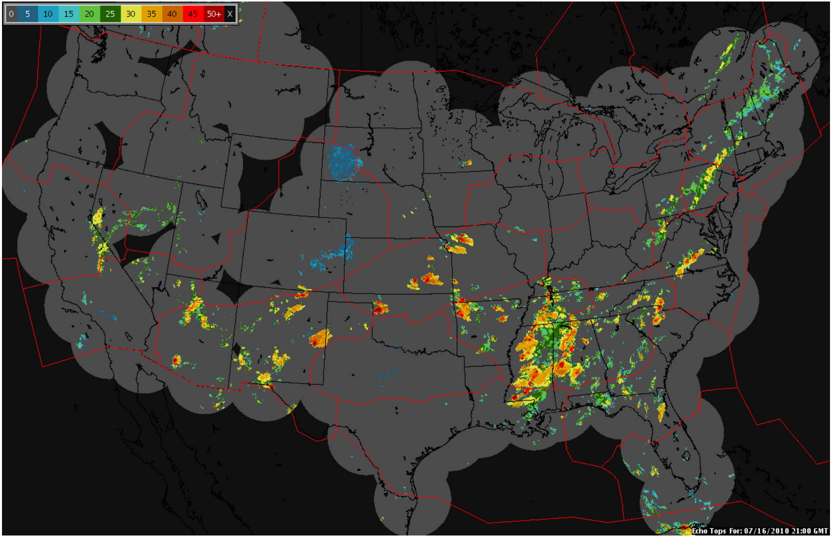

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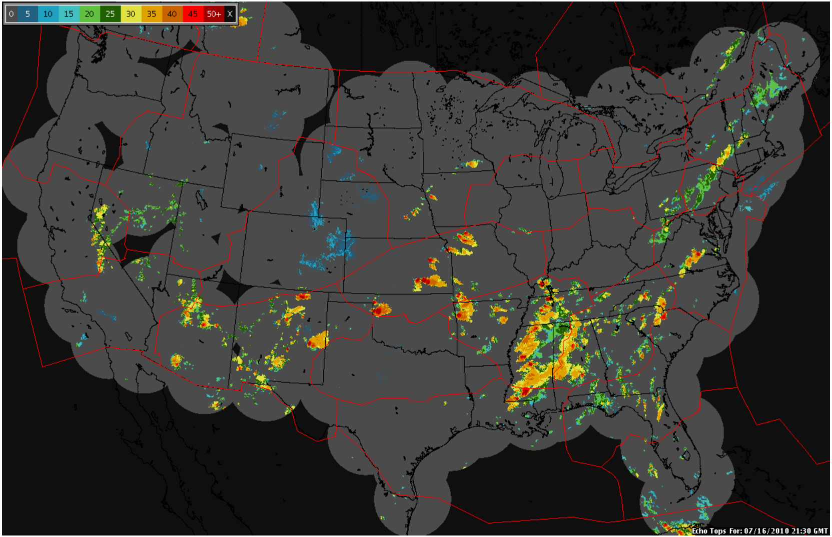

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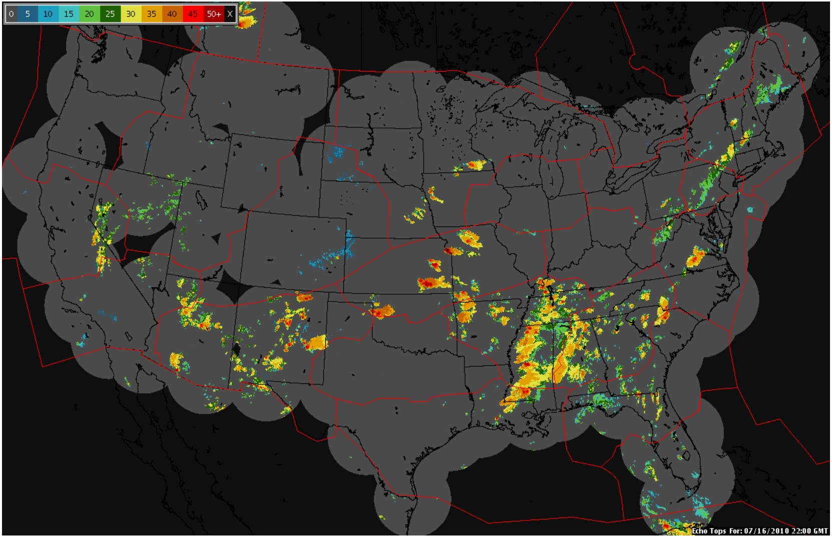

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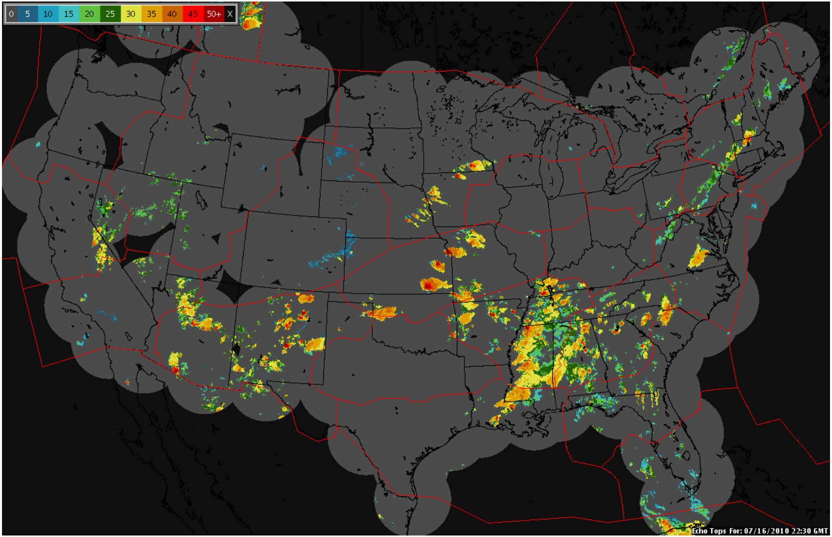

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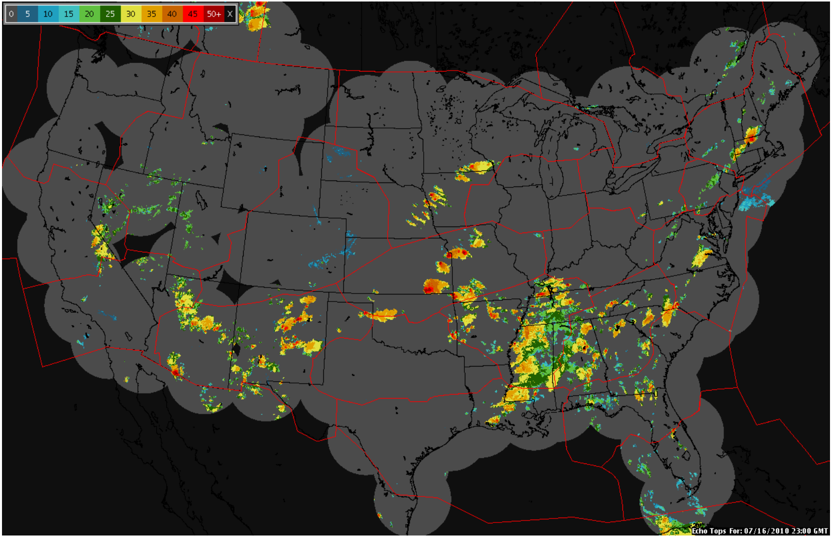
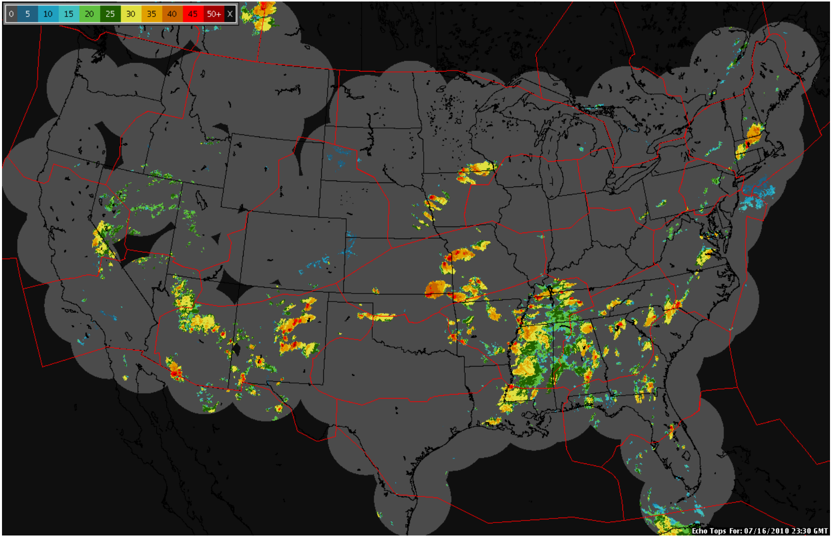


Echo Tops for July 16th, 2010



Preprocess data: 19m:48s

- Estimate airport capacities from APM : 01:53
- Get arrival and departure times from ASPM: 01:15
- Construct flight graphs: 09:07
- Find connecting flights from RITA: 03:07
- Adjust sector capacities using SDAT: 02:30

Performance analysis

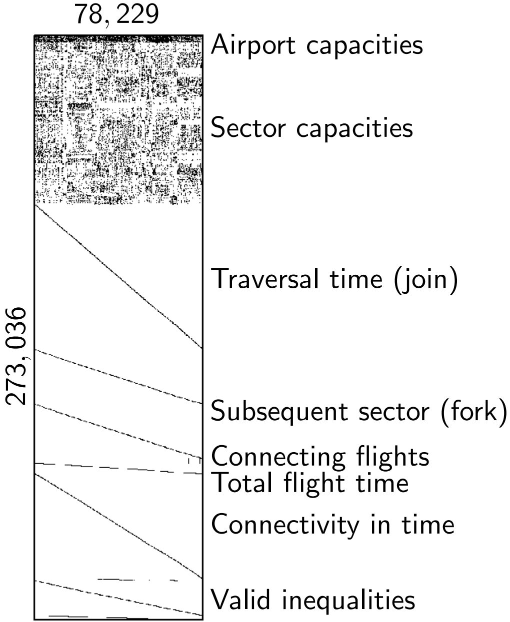
Preprocess data: 19m:48s

- Estimate airport capacities from APM : 01:53
- Get arrival and departure times from ASPM: 01:15
- Construct flight graphs: 09:07
- Find connecting flights from RITA: 03:07
- Adjust sector capacities using SDAT: 02:30

Form and solve model: 3m:52s

- Process data (convert time to discrete model time): 0:45
- Form constraint matrix and objective: 2:44
- Solve optimization problem: 0:18

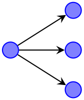
$$\begin{aligned} \underset{w}{\text{minimize}} \quad & c^T w \\ \text{subject to} \quad & Aw \le b \\ & w \in \{0,1\}^n \end{aligned}$$

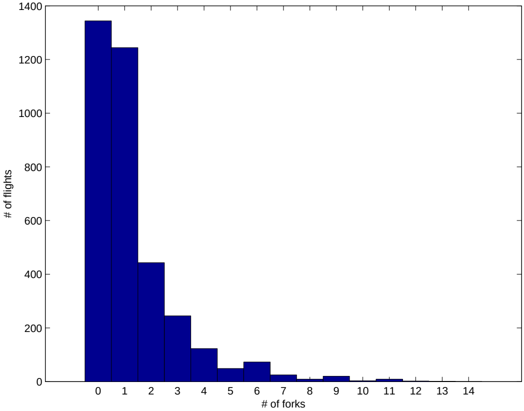


```
Optimize a model with 273036 rows, 156458 columns and 646381 nonzeros
Presolve removed 240751 rows and 136602 columns
Presolve time: 2.87s
Presolved: 32285 rows, 19856 columns, 77701 nonzeros
Variable types: 0 continuous, 19856 integer (19854 binary)
Found heuristic solution: objective 46550.000000

Root relaxation: objective 4.548444e+04, 13240 iterations, 0.22 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 45484.4420    0 6090 46550.0000 45484.4420  2.29%     -    3s
H    0     0                    46001.000000 45484.4420  1.12%     -    6s
H    0     0                    45958.000000 45484.4420  1.03%     -    6s
     0     0 45747.9552    0 2797 45958.0000 45747.9552  0.46%     -    7s
H    0     0                    45888.000000 45747.9552  0.31%     -    7s
     0     0 45772.1095    0 2226 45888.0000 45772.1095  0.25%     -    8s
H    0     0                    45864.000000 45772.1095  0.20%     -    8s
     0     0 45782.3585    0 1812 45864.0000 45782.3585  0.18%     -    8s
H    0     0                    45846.000000 45782.3585  0.14%     -    8s
H    0     0                    45820.000000 45782.3585  0.08%     -    9s
     .     .                              .          .       .      .     .
     .     .                              .          .       .      .     .
     .     .                              .          .       .      .     .
     0     0 45791.0917    0 1356 45812.0000 45791.0917  0.05%     -   12s
     0     0 45791.0974    0 1364 45812.0000 45791.0974  0.05%     -   12s
     0     0 45791.0974    0 1364 45812.0000 45791.0974  0.05%     -   12s
H    0     0                    45811.000000 45791.0974  0.04%     -   16s
H    0     0                    45809.000000 45791.0974  0.04%     -   16s

Explored 0 nodes (35857 simplex iterations) in 17.50 seconds
Thread count was 4 (of 4 available processors)

Optimal solution found (tolerance 1.00e-04)
Best objective 4.580900000000e+04, best bound 4.580900000000e+04, gap 0.0%
```

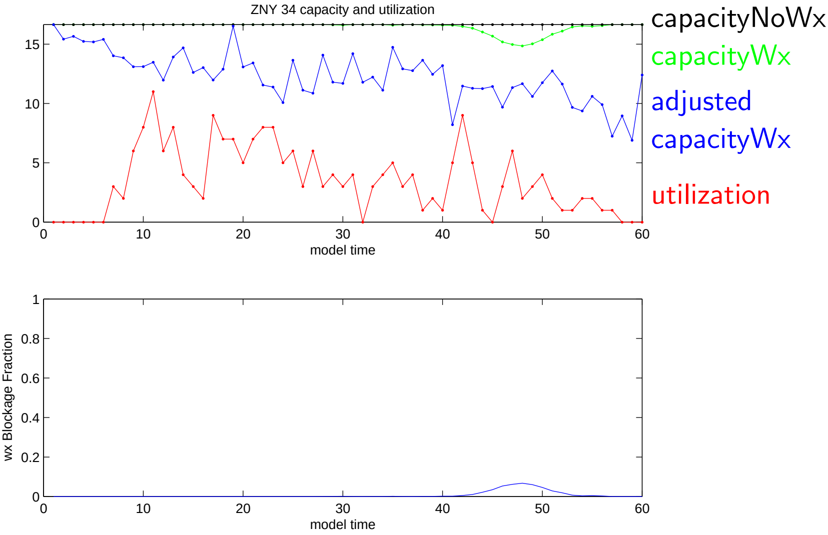
The number of forks

- The number of forks in a flight's directed graph is a proxy for the number of paths
- Most flights have only a few paths

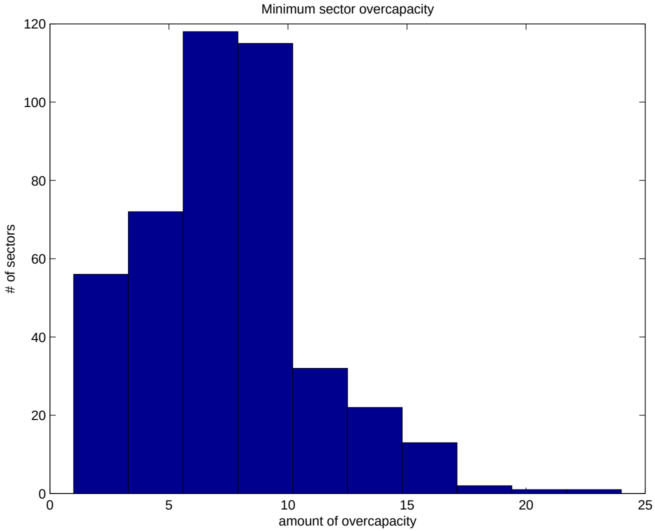




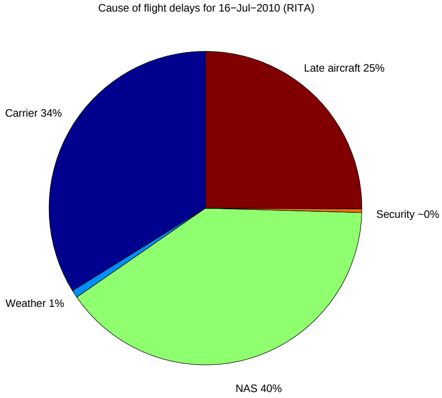
Sector utilization



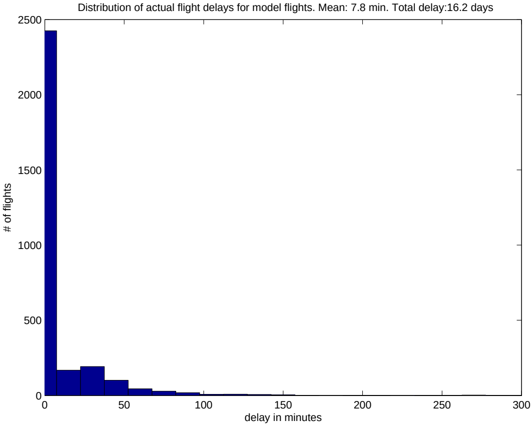
Sector overcapacity



- RITA contains information about amount and type of delay experienced by flights

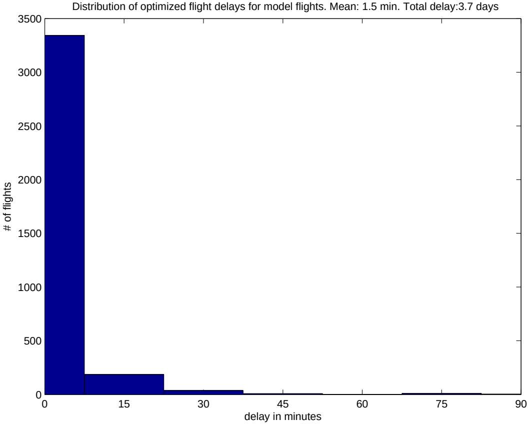


- RITA contains information about amount and type of delay experienced by flights
- Actual delay experienced (by model flights): 16.2 days



Flight Delays

- RITA contains information about amount and type of delay experienced by flights
- Actual delay experienced (by model flights): 16.2 days
- Optimal delay: 3.7 days



Conclusions and further work

Conclusions and further work

Conclusions:

- Preliminary results suggest model capable of NAS scale
- May be used to analyze impact of weather on air traffic

Further work:

- More days
- More airports, more flights, more super-high (or low) sectors
- Produce flight DAGs with more diverse paths
- Produce different flight DAGs for different aircraft types
- Visualization of flight schedules

  Currently...

```
Flight AAL1062:DFW-DCA 4:35 PM-7:39 PM time:03:04 delay:19
Flight AAL1062:DFW-DCA departs DFW    at  4:45 PM
Flight AAL1062:DFW-DCA enters  ZFW 90 at  5:00 PM
Flight AAL1062:DFW-DCA enters  ZME 44 at  5:30 PM
Flight AAL1062:DFW-DCA enters  ZME 26 at  6:00 PM
Flight AAL1062:DFW-DCA enters  ZME 62 at  6:15 PM
Flight AAL1062:DFW-DCA enters  ZID 86 at  6:45 PM
Flight AAL1062:DFW-DCA enters  ZDC 37 at  7:00 PM
Flight AAL1062:DFW-DCA arrives DCA    at  7:30 PM 02:45
```

- Better fidelity in TRACON

Thank you

Extra slides

Constructing directed acyclic flight graphs

Graph $G(V, E, W)$ formed from paths may contain cycles. Model requires an acyclic graph. Solve a minimum feedback arc set problem to remove edges from cycles while maintaining s-t path.

$$
\begin{aligned}
\text{minimize} \quad & \sum_{(i,j)\in E} x_{ij} w_{ij} \\
\text{subject to} \quad & \sum_{(i,j)\in C} x_{ij} \geq 1, \quad \forall C \in \mathcal{C}, \\
& \sum_{u:(i,u)\in E} y_{iu} - \sum_{v:(v,i)\in E} y_{vi} = \begin{cases} 1 & i = s \\ -1 & i = t \\ 0 & \text{otherwise} \end{cases}, \quad \forall i \in V, \\
& y_{ij} \leq 1 - x_{ij}, \quad \forall (i,j) \in E, \\
& x_{ij} \in \{0,1\}, \quad \forall (i,j) \in E, \\
& y_{ij} \in \{0,1\}, \quad \forall (i,j) \in E,
\end{aligned}
$$

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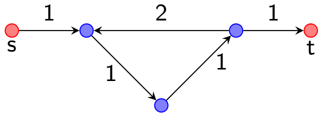

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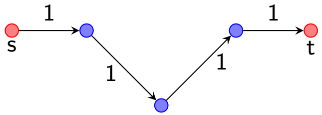

- Ngaire Underhill preprocessed ETMS data and provided

```
FLIGHT_ID ORIG    DEST
FLIGHT_ID ORIG    DEPART_TIME
FLIGHT_ID SECTOR1 ENTRANCE_TIME1
...       ...     ...
FLIGHT_ID SECTORN ENTRANCE_TIMEN
FLIGT_ID  DEST    ARRIVE_TIME
```

- Let T_j^f be the observed entrance time of sector j for flight f
- Let $L_{jj'}^f = T_{j'}^f - T_j^f$ be the observed transit time from sector j to j' for flight f
- For all flights f with the same origin and destination, we compute:
  - μ and σ the sample mean and variance of $\{T_j^f\}_f$
  - First entrance estimate: $\underline{T}_j^f = \max(\min_f(T_j^f), \mu - 2\sigma)$
  - Last entrance estimate: $\overline{T}_j^f = \min(\max_f(T_j^f), \mu + 2\sigma)$
  - Transit time estimate: $l_{jj'}^f = \min_f(L_{jj'}^f)$